Mathematics overview: Step 4

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• Solve two linear simultaneous equations algebraically and graphically
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Calculating

12 hours

Key concepts

The Big Picture: Calculation progression map

- calculate with roots, and with integer indices
- calculate with standard form $A \times 10^n$, where $1 \le A < 10$ and n is an integer
- use inequality notation to specify simple error intervals due to truncation or rounding
- apply and interpret limits of accuracy

Return to overview

Possible learning intentions

- Calculate with powers and roots
- Explore the use of standard form
- Explore the effects of rounding

Possible success criteria

- Calculate with positive indices (roots) using written methods
- Calculate with negative indices in the context of standard form
- Use a calculator to evaluate numerical expressions involving powers (roots)
- Interpret a number written in standard form
- Add (subtract) numbers written in standard form
- Multiply (divide) numbers written in standard form
- Convert a 'near miss' into standard form; e.g. 23×10^7
- Enter a calculation written in standard form into a scientific calculator
- Interpret the standard form display of a scientific calculator
- Understand the difference between truncating and rounding
- Identify the minimum and maximum values of an amount that has been rounded (to nearest x, x d.p., x s.f.)
- Use inequalities to describe the range of values for a rounded value
- Solve problems involving the maximum and minimum values of an amount that has been rounded

Prerequisites

- Know the meaning of powers
- Know the meaning of roots
- Know the multiplication and division laws of indices
- Understand and use standard form to write numbers
- Round to a given number of decimal places or significant figures
- Know the meaning of the symbols $<, >, \le, \ge$

Mathematical language

Power
Root
Index, Indices
Standard form
Inequality
Truncate
Round
Minimum, Maximum
Interval
Decimal place
Significant figure

Notation

Standard form: $A \times 10^n$, where $1 \le A < 10$ and n is an integer
Inequalities: e.g. $x > 3$, $-2 < x \le 5$

Pedagogical notes

Liaise with the science department to establish when students first meet the use of standard form, and in what contexts they will be expected to interpret it.
NCETM: Departmental workshops: Index Numbers
NCETM: Glossary

Common approaches

The description 'standard form' is always used instead of 'scientific notation' or 'standard index form'.
Standard form is used to introduce the concept of calculating with negative indices. The link between 10^{-n} and $1/10^n$ can be established.
The language 'negative number' is used instead of 'minus number'.

Reasoning opportunities and probing questions

- Kenny thinks this number is written in standard form: 23×10^7. Do you agree with Kenny? Explain your answer.
- When a number 'x' is rounded to 2 significant figures the result is 70. Jenny writes '$65 < x < 75$'. What is wrong with Jenny's statement? How would you correct it?
- Convince me that $4.5 \times 10^7 \times 3 \times 10^5 = 1.35 \times 10^{13}$

Suggested activities

KM: Maths to Infinity: Standard form
KM: Maths to Infinity: Indices
Investigate 'Narcissistic Numbers'.
NRICH: Power mad!
NRICH: A question of scale
The scale of the universe animation (external site)

Learning review

www.diagnosticquestions.com

Possible misconceptions

- Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form
- When rounding to significant figures some students may think, for example, that 6729 rounded to one significant figure is 7
- Some students may struggle to understand why the maximum value of a rounded number is actually a value which would not round to that number; i.e. if given the fact that a number 'x' is rounded to 2 significant figures the result is 70, they might write '$65 < x < 74.99$'

Visualising and constructing — *10 hours*

Key concepts

The Big Picture: [Properties of Shape progression map]

- use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle)
- use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
- construct plans and elevations of 3D shapes

Return to overview

Possible learning intentions

- Know standard mathematical constructions
- Apply standard mathematical constructions
- Explore ways of representing 3D shapes

Possible success criteria

- Use compasses to construct clean arcs
- Use ruler and compasses to construct the perpendicular bisector of a line segment
- Use ruler and compasses to bisect an angle
- Use a ruler and compasses to construct a perpendicular to a line from a point (at a point)
- Understand the meaning of locus (loci)
- Know how to construct the locus of points a fixed distance from a point (from a line)
- Identify when to use the locus of points a fixed distance from a point (from a line)
- Identify when a perpendicular bisector is needed to solve a loci problem
- Identify when an angle bisector is needed to solve a loci problem
- Choose techniques to construct 2D shapes; e.g. rhombus
- Combine techniques to solve more complex loci problems
- Know how to deal with a change in depth when dealing with plans and elevations
- Construct a shape from its plans and elevations
- Construct the plan and elevations of a given shape

Prerequisites

- Measure distances to the nearest millimetre
- Create and interpret scale diagrams
- Use compasses to draw circles
- Interpret plan and elevations

Mathematical language

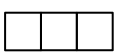
Compasses
Arc
Line segment
Perpendicular
Bisect
Perpendicular bisector
Locus, Loci
Plan
Elevation

Pedagogical notes

Ensure that students always leave their construction arcs visible. Arcs must be 'clean'; i.e. smooth, single arcs with a sharp pencil.
NCETM: [Departmental workshops: Constructions]
NCETM: [Departmental workshops: Loci]
NCETM: [Glossary]

Common approaches
All pupils should experience using dynamic software (e.g. Autograph) to explore standard mathematical constructions (perpendicular bisector and angle bisector).

Reasoning opportunities and probing questions

- (Given a single point marked on the board) show me a point 30 cm away from this point. And another. And another …
- Provide shapes made from some cubes in certain orientations. Challenge pupils to construct the plans and elevations. Do groups agree?
- If this is the plan  show me a possible 3D Shape. And another. And another.
- Demonstrate how to create the perpendicular bisector (or other constructions). Challenge pupils to write a set of instructions for carrying out the construction. Follow these instructions very precisely (being awkward if possible; e.g. changing radius of compasses). Do the instructions work?
- Give pupils the equipment to create standard constructions and challenge them to create a right angle / bisect an angle

Suggested activities

KM: [Construction instruction]
KM: [Construction challenges]
KM: [Napoleonic challenge]
KM: [Circumcentre etcetera]
KM: [Locus hocus pocus]
KM: [The perpendicular bisector]
KM: [Topple]
KM: [Gilbert goat]
KM: [An elevated position]
KM: [Solid problems] (plans and elevations)

Learning review
[www.diagnosticquestions.com]

Possible misconceptions

- When constructing the bisector of an angle some pupils may think that the intersecting arcs need to be drawn from the ends of the two lines that make the angle.
- When constructing a locus such as the set of points a fixed distance from the perimeter of a rectangle, some pupils may not interpret the corner as a point (which therefore requires an arc as part of the locus)
- The north elevation is the view of a shape from the north (the north face of the shape), not the view of the shape while facing north.

Algebraic proficiency: tinkering — *9 hours*

Key concepts

The Big Picture: Algebra progression map

- understand and use the concepts and vocabulary of identities
- know the difference between an equation and an identity
- simplify and manipulate algebraic expressions by expanding products of two binomials and factorising quadratic expressions of the form $x^2 + bx + c$
- argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments
- translate simple situations or procedures into algebraic expressions or formulae

Return to overview

Possible learning intentions

- Understand equations and identities
- Manipulate algebraic expressions
- Construct algebraic statements

Possible success criteria

- Understand the meaning of an identity
- Multiply two linear expressions of the form $(x + a)(x + b)$
- Multiply two linear expressions of the form $(x \pm a)(x \pm b)$
- Expand the expression $(x \pm a)^2$
- Simplify an expression involving 'x^2' by collecting like terms
- Identify when it is necessary to remove factors to factorise a quadratic expression
- Identify when it is necessary to find two linear expressions to factorise a quadratic expression
- Factorise a quadratic expression of the form $x^2 + bx + c$
- Know how to set up an mathematical argument
- Work out why two algebraic expressions are equivalent
- Create a mathematical argument to show that two algebraic expressions are equivalent
- Identify variables in a situation
- Distinguish between situations that can be modelled by an expression or a formula
- Create an expression or a formula to describe a situation

Prerequisites

- Manipulate expressions by collecting like terms
- Know that $x \times x = x^2$
- Calculate with negative numbers
- Know the grid method for multiplying two two-digit numbers
- Know the difference between an expression, an equation and a formula

Mathematical language

Inequality
Identity
Equivalent
Equation
Formula, Formulae
Expression
Expand
Linear
Quadratic

Notation
The equals symbol '=' and the equivalency symbol '≡'

Pedagogical notes

Pupils should be taught to use the equivalency symbol '≡' when working with identities.
During this unit pupils could construct (and solve) equations in addition to expressions and formulae.
See former coursework task, opposite corners
NCETM: Algebra
NCETM: Departmental workshops: Deriving and Rearranging Formulae
NCETM: Glossary

Common approaches
All students are taught to use the grid method to multiply two linear expressions. They then use the same approach in reverse to factorise a quadratic.

Reasoning opportunities and probing questions

- The answer is $x^2 + 10x + c$. Show me a possible question. And another. And another … (Factorising a quadratic expression of the form $x^2 + bx + c$ can be introduced as a reasoning activity: once pupils are fluent at multiplying two linear expressions they can be asked 'if this is the answer, what is the question?')
- Convince me that $(x + 3)(x + 4)$ does not equal $x^2 + 7$.
- What is wrong with this statement? How can you correct it? $(x + 3)(x + 4) \equiv x^2 + 12x + 7$.
- Jenny thinks that $(x – 2)^2 = x^2 – 4$. Do you agree with Jenny? Expaloin your answer.

Suggested activities

KM: Stick on the Maths: Multiplying linear expressions
KM: Maths to Infinity: Brackets
KM: Maths to Infinity: Quadratics
NRICH: Pair Products
NRICH: Multiplication Square
NRICH: Why 24?

Learning review
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Possible misconceptions

- Once pupils know how to factorise a quadratic expression of the form $x^2 + bx + c$ they might overcomplicate the simpler case of factorising an expression such as $x^2 + 2x$ ($\equiv (x + 0)(x + 2)$)
- Many pupils may think that $(x + a)^2 \equiv x^2 + a^2$
- Some pupils may think that, for example, $-2 \times -3 = -6$
- Some pupils may think that $x^2 + 12 + 7x$ is not equivalent to $x^2 + 7x + 12$, and therefore think that they are wrong if the answer is given as $x^2 + 7x + 12$

Proportional reasoning — 9 hours

Key concepts

The Big Picture: Ratio and Proportion progression map

- solve problems involving direct and inverse proportion including graphical and algebraic representations
- apply the concepts of congruence and similarity, including the relationships between lengths in similar figures
- change freely between compound units (e.g. density, pressure) in numerical and algebraic contexts
- use compound units such as density and pressure

Return to overview

Possible learning intentions

- Solve problems involving different types of proportion
- Investigate ways of representing proportion
- Understand and solve problems involving congruence
- Understand and solve problems involving similarity
- Know and use compound units in a range of situations

Possible success criteria

- Know the difference between direct and inverse proportion
- Recognise direct (inverse) proportion in a situation
- Know the features of a graph that represents a direct (inverse) proportion situation
- Know the features of an expression (or formula) that represents a direct (inverse) proportion situation
- Understand the connection between the multiplier, the expression and the graph
- Know the meaning of congruent (similar) shapes
- Identify congruence (similarity) of shapes in a range of situations
- Identify the information required to solve a problem involving similar shapes
- Finding missing lengths in similar shapes
- Understand why speed, density and pressure are known as compound units
- Know the definition of density (pressure, population density, speed)
- Solve problems involving density (pressure, speed)
- Convert between units of density

Prerequisites

- Find a relevant multiplier in a situation involving proportion
- Plot the graph of a linear function
- Understand the meaning of a compound unit
- Convert between units of length, capacity, mass and time

Mathematical language

Direct proportion
Inverse proportion
Multiplier
Linear
Congruent, Congruence
Similar, Similarity
Compound unit
Density, Population density
Pressure

Notation

Kilograms per metre cubed is written as kg/m^3

Pedagogical notes

Pupils have explored enlargement previously.
Use the story of Archimedes and his 'eureka moment' when introducing density.
Up-to-date information about population densities of counties and cities of the UK, and countries of the world, is easily found online.
NCETM: The Bar Model
NCETM: Multiplicative reasoning
NCETM: Departmental workshops: Proportional Reasoning
NCETM: Departmental workshops: Congruence and Similarity
NCETM: Glossary

Common approaches

All pupils are taught to set up a 'proportion table' and use it to find the multiplier in situations involving direct proportion

Reasoning opportunities and probing questions

- Show me an example of two quantities that will be in direct (inverse) proportion. And another. And another …
- Convince me that this information shows a proportional relationship. What type of proportion is it?

40	3
60	2
80	1.5

- Which is the greatest density: $0.65g/cm^3$ or $650kg/m^3$? Convince me.

Suggested activities

KM: Graphing proportion
NRICH: In proportion
NRICH: Ratios and dilutions
NRICH: Similar rectangles
NRICH: Fit for photocopying
NRICH: Tennis
NRICH: How big?

Learning review

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Possible misconceptions

- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to solve problems
- The word 'similar' means something much more precise in this context than in other contexts pupils encounter. This can cause confusion.
- Some pupils may think that a multiplier always has to be greater than 1

Pattern sniffing — *8 hours*

Key concepts

The Big Picture: Algebra progression map

- recognise and use Fibonacci type sequences, quadratic sequences

Return to overview

Possible learning intentions

- Investigate Fibonacci numbers
- Investigate Fibonacci type sequences
- Explore quadratic sequences

Possible success criteria

- Recognise Fibonacci numbers
- Recognise the Fibonacci sequence
- Generate Fibonacci type sequences
- Find the next three terms in any Fibonacci type sequence
- Substitute numbers into formulae including terms in x^2
- Generate terms of a quadratic sequence
- Identify quadratic sequences
- Establish the first and second differences of a quadratic sequence
- Find the next three terms in any quadratic sequence
- Find the term in x^2 for a quadratic sequence
- Compare the term in x^2 and the whole sequence
- Find the nth term of a sequence of the form $ax^2 + b$
- Find the nth term of a sequence of the form $ax^2 + bx + c$

Prerequisites

- Generate a linear sequence from its nth term
- Substitute positive numbers into quadratic expressions
- Find the nth term for an increasing linear sequence
- Find the nth term for an decreasing linear sequence

Mathematical language

Term
Term-to-term rule
Position-to-term rule
nth term
Generate
Linear
Quadratic
First (second) difference
Fibonacci number
Fibonacci sequence

Notation

T(n) is often used to indicate the 'nth term'

Pedagogical notes

The Fibonacci sequence consists of the Fibonacci numbers (1, 1, 2, 3, 5, …), while a Fibonacci type sequence is any sequence formed by adding the two previous terms to get the next term.

NCETM: Departmental workshops: Sequences
NCETM: Glossary

Common approaches

All students should use a spreadsheet to explore aspects of sequences during this unit. For example, this could be using formulae to continue a given sequence, to generate the first few terms of a sequence from an nth term as entered, or to find the missing terms in a Fibonacci sequence as in 'Fibonacci solver'.

Reasoning opportunities and probing questions

- A sequence has the first two terms 1, 2, … Show me a way to continue this sequence. And another. And another …
- A sequence has nth term $3n^2 + 2n - 4$. Jenny writes down the first three terms as 1, 12, 29. Kenny writes down the first three terms as 1, 36, 83. Who do agree with? Why? What mistake has been made?
- A sequence starts with the terms 6, 12, 20, 30, … Find the nth term for this sequence (i.e. $n^2 + 3n + 2$). Look for patterns in how each of the numbers can be constructed. Is there another way to find the nth term (i.e. $(n+1)(n+2)$)? Show that the two nth terms are equivalent.

Suggested activities

KM: Forming Fibonacci equations
KM: Mathematician of the Month: Fibonacci
KM: Leonardo de Pisa
KM: Fibonacci solver. Pupils can be challenged to create one of these.
KM: Sequence plotting. A grid for plotting nth term against term.
KM: Maths to Infinity: Sequences
KM: Stick on the Maths: Quadratic sequences
NRICH: Fibs

Learning review

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Possible misconceptions

- Some students may think that it is possible to find an nth term for any sequence. A Fibonacci type sequence would require a recurrence relation instead.
- Some students may think that the second difference (of a quadratic sequence) is equivalent to the coefficient of x^2.
- Some students may substitute into ax^2 incorrectly, working out $(ax)^2$ instead.

Solving equations and inequalities I — 5 hours

Key concepts

The Big Picture: Algebra progression map

- understand and use the concepts and vocabulary of inequalities
- solve linear inequalities in one variable
- represent the solution set to an inequality on a number line

Return to overview

Possible learning intentions

- Explore the meaning of an inequality
- Solve linear inequalities

Possible success criteria

- Understand the meaning of the four inequality symbols
- Choose the correct inequality symbol for a particular situation
- Represent practical situations as inequalities
- Recognise a simple linear inequality
- Find the set of integers that are solutions to an inequality
- Use set notation to list a set of integers
- Use a formal method to solve an inequality
- Use a formal method to solve an inequality with unknowns on both sides
- Use a formal method to solve an inequality involving brackets
- Know how to deal with negative number terms in an inequality
- Know how to show a range of values that solve an inequality on a number line
- Know when to use an open circle at the end of a range of values shown on a number line
- Know when to use an filled circle at the end of a range of values shown on a number line
- Use a number line to find the set of values that are true for two inequalities

Prerequisites

- Understand the meaning of the four inequality symbols
- Solve linear equations including those with unknowns on both sides

Mathematical language

(Linear) inequality
Unknown
Manipulate
Solve
Solution set
Integer

Notation

The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to)
The number line to represent solutions to inequalities. An open circle represents a boundary that is not included. A filled circle represents a boundary that is included.
Set notation; e.g. {-2, -1, 0, 1, 2, 3, 4}

Pedagogical notes

The mathematical process of solving a linear inequality is identical to that of solving linear equations. The only exception is knowing how to deal with situations when multiplication or division by a negative number is a possibility. Therefore, take time to ensure pupils understand the concept and vocabulary of inequalities.
NCETM: Departmental workshops: Inequalities
NCETM: Glossary

Common approaches

Pupils are taught to manipulate algebraically rather than be taught 'tricks'. For example, in the case of -2x > 8, pupils should not be taught to flip the inequality when dividing by -2. They should be taught to add 2x to both sides. Many pupils themselves will later generalise.

Reasoning opportunities and probing questions

- Show me an inequality (with unknowns on both sides) with the solution $x \geq 5$. And another. And another …
- Convince me that there are only 5 common integer solutions to the inequalities $4x < 28$ and $2x + 3 \geq 7$.
- What is wrong with this statement? How can you correct it? $1 – 5x \geq 8x – 15$ so $1 \geq 3x – 15$.

Suggested activities

KM: Stick on the Maths: Inequalities
KM: Convinced?: Inequalities in one variable
NRICH: Inequalities

Learning review

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Possible misconceptions

- Some pupils may think that it is possible to multiply or divide both sides of an inequality by a negative number with no impact on the inequality (e.g. if $-2x > 12$ then $x > -6$)
- Some pupils may think that a negative x term can be eliminated by subtracting that term (e.g. if $2 – 3x \geq 5x + 7$, then $2 \geq 2x + 7$)
- Some pupils may know that a useful strategy is to multiply out any brackets, but apply incorrect thinking to this process (e.g. if $2(3x – 3) < 4x + 5$, then $6x – 3 < 4x + 5$)

Calculating space — *13 hours*

Key concepts

The Big Picture: [Measurement and mensuration progression map]

- identify and apply circle definitions and properties, including: tangent, arc, sector and segment
- calculate arc lengths, angles and areas of sectors of circles
- *calculate surface area of right prisms (including cylinders)*
- calculate exactly with multiples of π
- know the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$, and apply it to find lengths in right-angled triangles in two dimensional figures

Return to overview

Possible learning intentions

- Solve problems involving arcs and sectors
- Solve problems involving prisms
- Investigate right-angled triangles
- Solve problems involving Pythagoras' theorem

Possible success criteria

- Know the vocabulary of circles
- Know how to find arc length
- Calculate the arc length of a sector when radius is given
- Know how to find the area of a sector
- Calculate the area of a sector when radius is given
- Calculate the angle of a sector when the arc length and radius are known
- Know how to find the surface area of a right prism (cylinder)
- Calculate the surface area of a right prism (cylinder)
- Calculate exactly with multiples of π
- Know Pythagoras' theorem
- Identify the hypotenuse in a right-angled triangle
- Know when to apply Pythagoras' theorem
- Calculate the hypotenuse of a right-angled triangle using Pythagoras' theorem
- Calculate one of the shorter sides in a right-angled triangle using Pythagoras' theorem

Prerequisites

- Know and use the number π
- Know and use the formula for area and circumference of a circle
- Know how to use formulae to find the area of rectangles, parallelograms, triangles and trapezia
- Know how to find the area of compound shapes

Mathematical language

Circle, Pi
Radius, diameter, chord, circumference, arc, tangent, sector, segment
(Right) prism, cylinder
Cross-section
Hypotenuse
Pythagoras' theorem

Notation

π
Abbreviations of units in the metric system: km, m, cm, mm, mm^2, cm^2, m^2, km^2, mm^3, cm^3, km^3

Pedagogical notes

This unit builds on the area and circle work form Stages 7 and 8. Pupils will need to be reminded of the key formula, in particular the importance of the perpendicular height when calculating areas and the correct use of πr^2. Note: some pupils may only find the area of the three 'distinct' faces when finding surface area.
Pupils must experience right-angled triangles in different orientations to appreciate the hypotenuse is always opposite the right angle.
NCETM: [Glossary]
Common approaches
Pupils visualize and write down the shapes of all the faces of a prism before calculating the surface area. Every classroom has a set of [area posters] on the wall.
Pythagoras' theorem is stated as 'the square of the hypotenuse is equal to the sum of the squares of the other two sides' not just $a^2 + b^2 = c^2$.

Reasoning opportunities and probing questions

- Show me a sector with area 25π. And another. And another …
- Always/ Sometimes/ Never: The value of the volume of a prism is less than the value of the surface area of a prism.
- Always/ Sometimes/ Never: If $a^2 + b^2 = c^2$, a triangle with sides a, b and c is right angled.
- Kenny thinks it is possible to use Pythagoras' theorem to find the height of isosceles triangles that are not right- angled. Do you agree with Kenny? Explain your answer.
- Convince me the hypotenuse can be represented as a horizontal line.

Suggested activities

KM: [The language of circles]
KM: [One old Greek] (geometrical derivation of Pythagoras' theorem. This is explored further in the next unit)
KM: [Stick on the Maths: Pythagoras' Theorem]
KM: [Stick on the Maths: Right Prisms]
NRICH: [Curvy Areas]
NRICH: [Changing Areas, Changing Volumes]

Learning review
www.diagnosticquestions.com

Possible misconceptions

- Some pupils will work out $(\pi \times r)^2$ when finding the area of a circle
- Some pupils may use the sloping height when finding cross-sectional areas that are parallelograms, triangles or trapezia
- Some pupils may confuse the concepts of surface area and volume
- Some pupils may use Pythagoras' theorem as though the missing side is always the hypotenuse
- Some pupils may not include the lengths of the radii when calculating the perimeter of an arc

Conjecturing — 6 hours

Key concepts

The Big Picture: Properties of Shape progression map

- use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
- apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

Return to overview

Possible learning intentions

- Explore the congruence of triangles
- Investigate geometrical situations
- Form conjectures
- Create a mathematical proof

Possible success criteria

- Know the criteria for triangles to be congruent (SSS, SAS, ASA, RHS)
- Identify congruent triangles
- Use known facts to form conjectures about lines and angles in geometrical situations
- Use known facts to derive further information in geometrical situations
- Test conjectures using known facts
- Know the structure of a simple mathematical proof
- Use known facts to create simple proofs
- Explain why the base angles in an isosceles triangle must be equal
- Explain the connections between Pythagorean triples

Prerequisites

- Know angle facts including angles at a point, on a line and in a triangle
- Know angle facts involving parallel lines and vertically opposite angles
- Know the properties of special quadrilaterals
- Know Pythagoras' theorem

Mathematical language

Congruent, congruence
Similar (shapes), similarity
Hypotenuse
Conjecture
Derive
Prove, proof
Counterexample

Notation

Notation for equal lengths and parallel lines
SSS, SAS, ASA, RHS
The 'implies that' symbol (⇒)

Pedagogical notes

'Known facts' should include angle facts, triangle congruence, similarity and properties of quadrilaterals
NCETM: Glossary

Common approaches

All students are asked to draw 1, 2, 3 and 4 points on the circumference of a set of circles. In each case, they join each point to every other point and count the number of regions the circle has been divided into. Using the results 1, 2, 4 and 8 they form a conjecture that the sequence is the powers of 2. They test this conjecture for the case of 5 points and find the circle is divided into 16 regions as expected. Is this enough to be convinced? It turns out that it should not be, as 6 points yields either 30 or 31 regions depending on how the points are arranged. This example is used to emphasise the importance and power of mathematical proof. See KM: Geometrical proof

Reasoning opportunities and probing questions

- Show me a pair of congruent triangles. And another. And another
- Show me a pair of similar triangles. And another. And another
- What is the same and what is different: Proof, Conjecture, Justification, Test?
- Convince me the base angles of an isosceles triangle are equal.
- Show me a Pythagorean Triple. And another. And another.
- Convince me a triangle with sides 3, 4, 5 is right-angled but a triangle with sides 4, 5, 6 is not right-angled.

Suggested activities

KM: Geometrical proof
KM: Shape work: Triangles to thirds, 4×4 square, Squares, Congruent triangles
KM: Daniel Gumb's cave
KM: Pythagorean triples
KM: Stick on the Maths: Congruence and similarity
NRICH: Tilted squares
NRICH: What's possible?

Learning review

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Possible misconceptions

- Some pupils think AAA is a valid criterion for congruent triangles.
- Some pupils try and prove a geometrical situation using facts that 'look OK', for example, *'angle ABC looks like a right angle'*.
- Some pupils do not appreciate that diagrams are often drawn to scale.
- Some pupils think that all triangles with sides that are consecutive numbers are right angled.

Algebraic proficiency: visualising — *12 hours*

Key concepts

The Big Picture: Algebra progression map

- identify and interpret gradients and intercepts of linear functions algebraically
- use the form $y = mx + c$ to identify parallel lines
- find the equation of the line through two given points, or through one point with a given gradient
- interpret the gradient of a straight line graph as a rate of change
- recognise, sketch and interpret graphs of quadratic functions
- recognise, sketch and interpret graphs of simple cubic functions and the reciprocal function $y = 1/x$ with $x \neq 0$
- plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

Return to overview

Possible learning intentions

- Investigate features of straight line graphs
- Explore graphs of quadratic functions
- Explore graphs of other standard non-linear functions
- Create and use graphs of non-standard functions
- Solve kinematic problems

Possible success criteria

- Use the form $y = mx + c$ to identify parallel lines
- Rearrange an equation into the form $y = mx + c$
- Find the equation of a line through one point with a given gradient
- Find the equation of a line through two given points
- Interpret the gradient of a straight line graph as a rate of change
- Plot graphs of quadratic (cubic, reciprocal) functions
- Recognise and interpret the graphs of quadratic (cubic, reciprocal) functions
- Sketch graphs of quadratic (cubic, reciprocal) functions
- Plot and interpret graphs of non-standard functions in real contexts
- Find approximate solutions to kinematic problems involving distance, speed and acceleration

Prerequisites

- Plot straight-line graphs
- Interpret gradients and intercepts of linear functions graphically and algebraically
- Recognise, sketch and interpret graphs of linear functions
- Recognise graphs of simple quadratic functions
- Plot and interpret graphs of kinematic problems involving distance and speed

Mathematical language

Function, equation
Linear, non-linear
Quadratic, cubic, reciprocal
Parabola, Asymptote
Gradient, y-intercept, x-intercept, root
Rate of change
Sketch, plot
Kinematic
Speed, distance, time
Acceleration, deceleration

Notation

$y = mx + c$

Pedagogical notes

This unit builds on the graphs of linear functions and simple quadratic functions work from Stage 8.
Where possible, students should be encouraged to plot linear graphs efficiently by using knowledge of the *y*-intercept and the gradient.
NCETM: Glossary

Common approaches

'Monter' and 'commencer' are shared as the reason for 'm' and 'c' in y = mx + c and links to y = ax + b
Students plot points with a 'x' and not '•'
Students draw graphs in pencil
All student use dynamic graphing software to explore graphs

Reasoning opportunities and probing questions

- Convince me the lines $y = 3 + 2x$, $y – 2x = 7$, $2x + 6 = y$ and $8 + y – 2x = 0$ are parallel to each other.
- What is the same and what is different: $y = x$, $y = x^2$, $y = x^3$ and $y=1/x$?
- Show me a sketch of a quadratic (cubic, reciprocal) graph. And another. And another …
- Sketch a distance/time graph of your journey to school. What is the same and what is different with the graph of a classmate?

Suggested activities

KM: Screenshot challenge
KM: Stick on the Maths: Quadratic and cubic functions
KM: Stick on the Maths: Algebraic Graphs
NRICH: Diamond Collector
NRICH: Fill me up
NRICH: What's that graph?
NRICH: Speed-time at the Olympics
NRICH: Exploring Quadratic Mappings
NRICH: Minus One Two Three

Learning review

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Possible misconceptions

- Some pupils do not rearrange the equation of a straight line to find the gradient of a straight line. For example, they think that the line $y – 2x = 6$ has a gradient of -2.
- Some pupils may think that gradient = (change in x) / (change in y) when trying to equation of a line through two given points.
- Some pupils may incorrectly square negative values of x when plotting graphs of quadratic functions.
- Some pupils think that the horizontal section of a distance time graph means an object is travelling at constant speed.
- Some pupils think that a section of a distance time graph with negative gradient means an object is travelling backwards or downhill.

Solving equations and inequalities II

8 hours

Key concepts

The Big Picture: Algebra progression map

- solve, in simple cases, two linear simultaneous equations in two variables algebraically
- derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- find approximate solutions to simultaneous equations using a graph

Return to overview

Possible learning intentions

- Solve simultaneous equations
- Use graphs to solve equations
- Solve problems involving simultaneous equations

Possible success criteria

- Understand that there are an infinite number of solutions to the equation $ax + by = c$ ($a \neq 0$, $b \neq 0$)
- Understand the concept of simultaneous equations
- Find approximate solutions to simultaneous equations using a graph
- Understand the concept of solving simultaneous equations by elimination*
- Target a variable to eliminate
- Decide if multiplication of one equation is required
- Decide whether addition or subtraction of equations is required
- Add or subtract pairs of equations to eliminate a variable
- Find the value of one variable in a pair of simple simultaneous equations
- Find the value of the second variable in a pair of simple simultaneous equations
- Solve two linear simultaneous equations in two variables in very simple cases (no multiplication required)
- Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required)
- Derive and solve two simultaneous equations
- Interpret the solution to a pair of simultaneous equations

Prerequisites

- Solve linear equations
- Substitute numbers into formulae
- Plot graphs of functions of the form $y = mx + c$, $x \pm y = c$ and $ax \pm by = c$)
- Manipulate expressions by multiplying by a single term

Mathematical language

Equation
Simultaneous equation
Variable
Manipulate
Eliminate
Solve
Derive
Interpret

Pedagogical notes

Pupils will be expected to solve simultaneous equations in more complex cases in Stage 10. This includes involving multiplications of both equations to enable elimination, cases where rearrangement is required first, and the method of substitution.
NCETM: Glossary

Common approaches

Pupils are taught to label the equations (1) and (2), and label the subsequent equation (3)
Teachers use graphs (i.e. dynamic software) to demonstrate solutions to simultaneous equations at every opportunity

Reasoning opportunities and probing questions

- Show me a solution to the equation $5a + b = 32$. And another, and another …
- Show me a pair of simultaneous equations with the solution $x = 2$ and $y = -5$. And another, and another …
- Kenny and Jenny are solving the simultaneous equations $x + 4y = 7$ and $x – 2y = 1$. Kenny thinks the equations should be added. Jenny thinks they should be subtracted. Who do you agree with? Explain why.

Suggested activities

KM: Stick on the Maths ALG2: Simultaneous linear equations
NRICH: What's it worth?
NRICH: Warmsnug Double Glazing
NRICH: Arithmagons

Learning review
www.diagnosticquestions.com

Possible misconceptions

- Some pupils may think that addition of equations is required when both equations involve a subtraction
- Some pupils may not multiply all coefficients, or the constant, when multiplying an equation
- Some pupils may think that it is always right to eliminate the first variable
- Some pupils may struggle to deal with negative numbers correctly when adding or subtracting the equations

Understanding risk — 8 hours

Key concepts

The Big Picture: Probability progression map

- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- enumerate sets and combinations of sets systematically, using tree diagrams
- understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size

Return to overview

Possible learning intentions

- Understand and use tree diagrams
- Develop understanding of probability in situations involving combined events
- Use probability to make predictions

Possible success criteria

- List outcomes of combined events using a tree diagram
- Label a tree diagram with probabilities
- Label a tree diagram with probabilities when events are dependent
- Know when to add two or more probabilities
- Know when to multiply two or more probabilities
- Use a tree diagram to calculate probabilities of independent combined events
- Use a tree diagram to calculate probabilities of dependent combined events
- Understand that relative frequency tends towards theoretical probability as sample size increases

Prerequisites

- Add fractions (decimals)
- Multiply fractions (decimals)
- Convert between fractions, decimals and percentages
- Use frequency trees to record outcomes of probability experiments
- Use experimental and theoretical probability to calculate expected outcomes

Mathematical language

Outcome, equally likely outcomes
Event, independent event, dependent event
Tree diagrams
Theoretical probability
Experimental probability
Random
Bias, unbiased, fair
Relative frequency
Enumerate
Set

Notation

P(A) for the probability of event A
Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words

Pedagogical notes

Tree diagrams can be introduced as simply an alternative way of listing all outcomes for multiple events. For example, if two coins are flipped, the possible outcomes can be listed (a) systematically, (b) in a two-way table, or (c) in a tree diagram. However, the tree diagram has the advantage that it can be extended to more than two events (e.g. three coins are flipped).
NCETM: Glossary

Common approaches

All students carry out the drawing pin experiment
Students are taught not to simply fractions when finding probabilities of combined events using a tree diagram (so that a simple check can be made that the probabilities sum to 1)

Reasoning opportunities and probing questions

- Show me an example of a probability problem that involves adding (multiplying) probabilities
- Convince me that there are eight different outcomes when three coins are flipped together
- Always / Sometimes / Never: increasing the number of times an experiment is carried out gives an estimated probability that is closer to the theoretical probability

Suggested activities

KM: Stick on the Maths: Tree diagrams
KM: Stick on the Maths: Relative frequency
KM: The drawing pin experiment

Learning review

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Possible misconceptions

- When constructing a tree diagram for a given situation, some students may struggle to distinguish between how events, and outcomes of those events, are represented
- Some students may muddle the conditions for adding and multiplying probabilities
- Some students may add the denominators when adding fractions

Presentation of data — 5 hours

Key concepts

The Big Picture: Statistics progression map

- interpret and construct tables, charts and diagrams, including tables and line graphs for time series data and know their appropriate use
- draw estimated lines of best fit; make predictions
- know correlation does not indicate causation; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

Return to overview

Possible learning intentions

- Construct and interpret graphs of time series
- Interpret a range of charts and graphs
- Interpret scatter diagrams
- Explore correlation

Possible success criteria

- Construct graphs of time series
- Interpret graphs of time series
- Construct and interpret compound bar charts
- Interpret a wider range of non-standard graphs and charts
- Understand that correlation does not indicate causation
- Interpret a scatter diagram using understanding of correlation
- Construct a line of best fit on a scatter diagram
- Use a line of best fit to estimate values
- Know when it is appropriate to use a line of best fit to estimate values

Prerequisites

- Know the meaning of discrete and continuous data
- Interpret and construct frequency tables
- Construct and interpret pictograms, bar charts, pie charts, tables, vertical line charts, histograms (equal class widths) and scatter diagrams

Mathematical language

Categorical data, Discrete data
Continuous data, Grouped data
Axis, axes
Time series
Compound bar chart
Scatter graph (scatter diagram, scattergram, scatter plot)
Bivariate data
(Linear) Correlation
Positive correlation, Negative correlation
Line of best fit
Interpolate
Extrapolate
Trend

Notation
Correct use of inequality symbols when labeling groups in a frequency table

Pedagogical notes

Lines of best fit on scatter diagrams are first introduced in Stage 9, although pupils may well have encountered both lines and curves of best fit in science by this time.
William Playfair, a Scottish engineer and economist, introduced the line graph for time series data in 1786.
NCETM: Glossary

Common approaches
As a way of recording their thinking, all students construct the appropriate horizontal and vertical line when using a line of best fit to make estimates.
In simple cases, students plot the 'mean of x' against the 'mean of y' to help locate a line of best fit.

Reasoning opportunities and probing questions

- Show me a compound bar chart. And another. And another.
- What's the same and what's different: correlation, causation?
- What's the same and what's different: scatter diagram, time series, line graph, compound bar chart?
- Convince me how to construct a line of best fit.
- Always/Sometimes/Never: A line of best fit passes through the origin

Suggested activities

KM: Stick on the Maths HD2: Frequency polygons and scatter diagrams

Learning review
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Possible misconceptions

- Some pupils may think that correlation implies causation
- Some pupils may think that a line of best fit always has to pass through the origin
- Some pupils may misuse the inequality symbols when working with a grouped frequency table